A PROCEDURE FOR EVALUATING CUMULATIVE DAMAGE IN FATIGUE

BY

FRANK EDWIN RICHART, JR.

B. S., University of Illinois, 1940

M. S., University of Illinois, 1946

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
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A PROCEDURE FOR EVALUATING CUMULATIVE DAMAGE IN FATIGUE

Introduction

There is at present quite a gap between the useful information available from tests concerning the failures of materials under repeated or "fatigue" stresses and the knowledge desired by designers of machine or structural elements. This is especially true when two or more stress levels above the endurance limit are applied for various numbers of cycles before failure, in designs based on some expected "life" of the material. Each overstress contributes to the ultimate failure of the material in some manner depending upon the magnitude of the overstress and the number of cycles. The manner in which the various stress cycles each contribute to failure is then the subject which needs further exploration.

It is the purpose of this thesis to investigate the stress-cycle relationships between various cycles of stresses above the endurance limit stress and to attempt to produce a theory which would explain their interactions. This theory should be such that it would provide information about the total "expected life" of a material subjected to various stress-cycle patterns. An hypothesis is presented in this thesis which apparently meets these conditions. Following the development of this hypothesis, tests were run to substantiate or disprove its usefulness.

Development of the Hypothesis

A review of previous work on the subjects of overstressing and understressing in fatigue has indicated that it is reasonable to assume that there exists some relation between the damage produced and the number of cycles applied at any stress during the progress of failure. Furthermore, it

appears that the damage begins with the first few cycles of overstress and that this process is continuous to failure. It has also been indicated that this damage-cycle relation is different for different levels of stress.

The above indications were utilized in the development of a method for the determination of the expected life of materials under multiple stress level types of loading. Two basic assumptions were made. They are: first, that some definite curve may represent the progress of failure of a material at a given stress level when this information is plotted on a damage (D) vs. cycle ration (R) diagram; and second, that any degree of damage applied to a material is unique, in that this same degree of damage may be applied by any of several magnitudes of stress if the proper number of cycles is used. Thus if we change the repeated stress level on a material from $\pm$ 40 000 psi to $\pm$ 50 000 psi we may represent this shift on the D-R diagram as a jump, along a constant damage line, from the curve representing the D-R relation for $\pm$ 40 000 psi to the curve representing the D-R relation for $\pm$ 50 000 psi.

To investigate the use of these D-R curves to represent the action of a multiple overstress test, it was convenient to assume a family of curves represented by the equation $D = R.^n$ All curves of this family pass through the points (0,0) and (1.0, 1.0) on the D-R diagram and thus conform to the physical conditions during a test that no damage has occurred at zero cycles and that 1.0 damage, or a fatigue crack has formed, when the cycle ratio is 1.0. Two curves were used for the initial investigation. An expression was derived for the value of cumulative cycle ratio obtained when infinitestimal numbers of cycles were applied alternately at each stress level, or along each D-R curve. This value of cumulative cycle ratio is a convenient notation for the total number of cycles applied, that is, the "life" of the material under this two-stress level type of test. When equal portions of the endurance lives at the two stresses were applied, the

value of cumulative cycle ratio obtained before failure was found to be less than 1.0. This value departed from 1.0 more when the two $D = R^n$ curves were farther apart than it did when the two curves were nearly identical. Also it was found that this value of cumulative cycle ratio was identical when determined from pairs of curves which were related by the ratio of n_2 to n_1 equal to a constant. n_2 and n_1 in the preceding equation, represent the exponents of the two curves used from the $D = R^n$ family.

Following the investigation of the $D = R^n$ family of curves, a number of more complex curves were studied to determine the value of cumulative cycle ratio obtained from their use. These complex curves showed results similar to those obtained from the $D = R^n$ curves. This analytical investigation indicated that only the relation between the cycle ratios at the various stress levels was significant in the determination of the value of cumulative damage and that it was not necessary to ascertain the exact D-R curve for each stress. Relative curves, or those which maintained the proper relations between the cycle ratios at the various stress levels, are therefore entirely satisfactory for further use.

These relative D-R curves may be determined experimentally by a two-stress level type of test if we assume any given curve to represent the D-R relation at the prestress. Then these relative curves may be used to compute the values of cumulative cycle ratio for various proportions of cycle ratio applied in any given pattern.

Specimens and Testing Machines Used

Two types of specimens were used in the experimental part of this investigation. Large plate specimens, 3½ in. wide and containing a 1 in. hole at the test section were cut from ½ in. A-7 killed structural steel plate. These specimens were roughed out, normalized for one half hour

at 1650 deg. F. to remove residual stresses from welding and machining, and then finish machined. They were tested in a lever type fatigue machine which applied a repeated axial load to the specimens. This machine was of the constant strain type and had a capacity of ± 50 000 lbs.

The small rotating beam specimens had a test section of 0.100 to 0.150 in. in diameter and were about 2 in. long. They were made from A-7 structural steel, normalized as above, and also from E.F. 4340 steel, heat treated at 1525 deg. F. and quenched in oil. They were tested in a Krouse High Speed Fatigue Testing Machine which applied a constant load to the end of a rotating cantlever beam having the test section of the specimen as its critical section.

The large fatigue machines needed frequent checking during each test and therefore these tests were interrupted at night. The small high speed fatigue machines ran continuously at 10 000 r. p. m. during each test except when it was necessary to stop to change a load setting during a test.

Results of Tests

Three types of tests were run to check the results of the analytical investigation. The first series was the usual constant stress level type of fatigue test which produced the information required for the basic S-N curves. This was done for all three sets of specimens.

The second series of test was the two-stress level or prestress-test stress type of test. One stress was selected as the prestress, and various numbers of cycles were applied to different specimens at this stress level. These specimens were then tested to failure at a second stress level. This type of test gave the information required to determine the relative curves for these two stress levels on the D-R diagram. This type of test was run on all three sets of specimens also.

The third series of tests was the alternate block type of test in which equal values of cycle ratio of two different stress levels were applied alternately to failure. This type of test gave information for the relations on the diagram of value of cumulative cycle ratio vs. size of block of cycle ratio. The results of these tests were then compared with the envelope curve which was obtained from the D-R diagrams by use of the theory mentioned in section II. This envelope curve formed a minimum boundary below which none of the test results of the large specimens fell. However the test results for one of the small E.F. 4340 specimens fell below this line. These small E.F. 4340 specimens showed rather an erratic behavior throughout all the tests due to a non-homogeneous grain structure or "banding" as found by metallurgical examination. The alternate block type of test was not run on the small specimens of A-7 steel.

These test results showed considerable scatter as might be expected. However, in all cases, a minimum curve could be determined on the S-N diagram, the D-R diagram, and on the value of cumulative cycle ratio vs. size of block of cycle ratio diagram. The minimum value curve, or the curve which indicated the maximum difference between the curves representing the two stress levels on the D-R diagram could be used for computations of the minimum values of cumulative cycle ratio. This computed value agreed rather well with the test values.

The results of all tests are given in tabular form as well as by curves. It is felt that the results of the large plate specimens were more reliable than the tests of the small specimens for several reasons. The A-7 killed steel was very uniform in grain structure, the first crack observed was considered as failure of the specimen, and this crack was easily identified when it occurred. In the tests of the small rotating beam specimens, failure was determined by rupture of the specimen and it was assumed that only a few cycles were involved during the period of crack propa-

gation, and finally the E.F. 4340 steel was found to be very inhomogeneous in grain structure. This caused unusual scatter in the S-N diagram, but in general the test values fell above the computed minimum curve for the values of cumulative cycle ratio.

Conclusions

This investigation indicated that it is reasonable to assume that a curve exists on the damage vs. cycle ratio diagram which represents the progress of failure at a given stress. This curve is continuous, and is different when it represents different levels of stress. These curves may be used in groups of two or more to determine the value of cumulative cycle ratio available when various amounts of cycle ratio are applied at each stress. The value of cumulative cycle ratio is a number which represents the life of the specimen, or the total number of cycles available at the different stresses before failure.

When two curves were used and small equal values of cycle ratio were applied alternately to failure, the value of cumulative cycle ratio obtained was always less than 1.0. Greater divergence of the two curves produced a larger reduction of this value from 1.0. When large equal values of cycle ratio were applied to two curves, the order in which the curves were used produced a significant effect on the value of cumulative cycle ratio, or the order in which the stresses were applied produced a noticeable effect on the life of the specimen when alternate blocks of more than about 3 per cent of the cycle ratios were used.

The value of cumulative cycle ratio was found to depend on the relative position of the curves on the D-R diagram rather than upon their exact position. The relative positions of the curves had to be such that the proper relation was maintained between the values of cycle ratio at the various stress levels involved. Using this fact that relative curves

were sufficient for further computations, a method was indicated whereby these relative D-R curves could be determined experimentally. After these experimentally determined curves were obtained, they could be used for calculation of the value of cumulative cycle ratio for any stress-cycle pattern desired.

The experimental results indicated that considerable scatter was encountered in the determination of these D-R curves. However, a minimum curve or maximum difference curve could be determined which would give minimum values for all further computations. This was desirable since designs are usually based on minimum values. The test results for the large plate specimens agreed well with the minimum curve on the value of cumulative cycle ratio vs. size of block of cycle ratio diagram. This minimum curve was obtained from computations based on the assumed curve and the maximum difference curve from the D-R diagram. The test results for the small specimens of E.F. 4340 steel showed more than the usual scatter. This was probably due to the inhomogeneity of the grain structure but in spite of this excess scatter, all but one of the test values fell above the computed minimum values of cumulative cycle ratio.

It appears that this procedure for combining the cumulative damaging effects, produced by various stress levels, to give an estimate of the life of the material is workable. However, the relative D-R curves must be determined experimentally as part of the procedure and hence this method cannot be used unless it is possible to run a number of tests to determine these D-R curves.

VITA

Frank Edwin Richart, Jr., was born in Urbana, Illinois, December 6, 1918. Upon graduation from Urbana High School he entered the University of Illinois, and received the degree of Bachelor of Science in Mechanical Engineering in June, 1940. The following September he entered the Graduate School of the University of Michigan at Ann Arbor, Michigan, to study Aeronautical Engineering.

In June, 1941, he was commissioned Ensign A-V (S) in the United States Naval Reserve and began active duty at the Naval Aircraft Factory in Philadelphia, Pennsylvania. His duties at the Naval Aircraft Factory included coordination of design, production, and acceptance testing, and finally laboratory testing of experimental type aircraft structures. He transferred from Philadelphia in December, 1944, and subsequently was connected with aircraft maintenance work in California. His last promotion was to Lieutenant Commander, USNR in October, 1945.

In November, 1945, upon beginning his terminal leave he entered the Graduate School of the University of Illinois, and received the Master of Science degree in Civil Engineering in September, 1946. Since that time, he has continued his graduate work at the university.

He was appointed Special Research Graduate Assistant in Civil Engineering in June, 1946, and Special Research Associate in Civil Engineering in June, 1947, which position he now holds.